

# Digital

22/2/2016

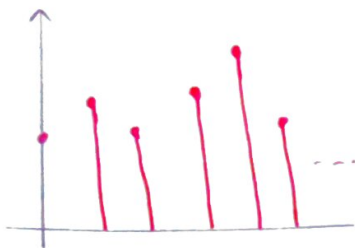
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د. عرفة

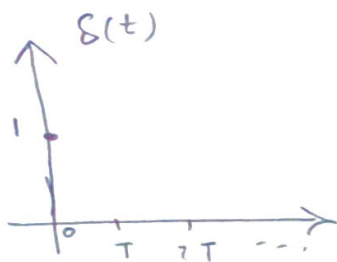
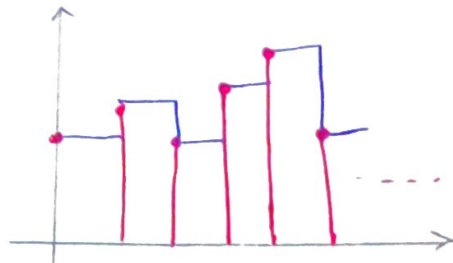
خافرة [2]

The T.F. of Z.O.H:-

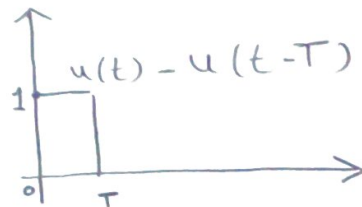
Z.O.H = Zero order hold



Z.O.H  $\Rightarrow$



Z.O.H  $\Rightarrow$



T.F. for Z.O.H =  $G_{Z.O.H} = G_h = \frac{O/P(s)}{I/P(s)}$

Laplace Transform  $\Rightarrow$

$$= \frac{L(u(t) - u(t-T))}{L(d(t))}$$

$$= \frac{1/s - \frac{e^{-Ts}}{s}}{1} = \frac{1}{s} (1 - e^{-Ts})$$

$$G_{Z.O.H}(s) = \frac{1 - e^{-Ts}}{s}$$

# Z.T  $G_h(s) \cdot G_1(s)$

$$= Z \left[ \frac{1 - e^{-Ts}}{s} G_1(s) \right]$$

T-domain  $\rightarrow$  Laplace  $\rightarrow$  Z.T

$$= (1 - z^{-1}) Z \left[ \frac{G_1(s)}{s} \right]$$

Z.T.

\* The Z.O.H gain = 1

$$\begin{aligned} Z.[G_h(s)] &= Z\left[1 - \frac{e^{-Ts}}{s}\right] \\ &= (1 - z^{-1}) Z\left[\frac{1}{s}\right] \\ &= \left(\frac{z^{-1}}{z}\right) \cdot \left(\frac{z}{z-1}\right) = 1 \end{aligned}$$

### # Pulse T.F.

The discrete or digital T.F. is called pulse T.F.

$$\text{Pulse T.F.} = \frac{C^*(s)}{R^*(s)} = \frac{C(z)}{R(z)}$$

① Pulse T.F. from difference equations

for ex:

$$y(k-1) + 2y(k) + 3y(k-2) = r(k)$$

Find the impulse T.F. =  $\frac{Y(z)}{R(z)}$

↓  
zT

$$z^{-1}Y(z) + 2Y(z) + 3z^{-2}Y(z) = R(z)$$

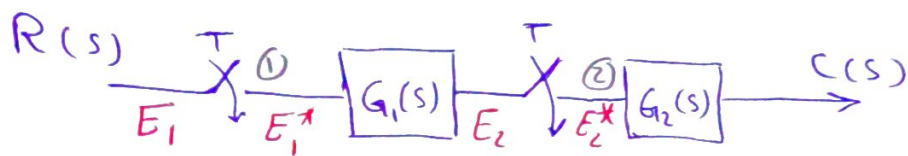
$$(3z^{-2} + z^{-1} + 2)Y(z) = R(z)$$

$$\Rightarrow \frac{Y(z)}{R(z)} = \frac{1}{3z^{-2} + z^{-1} + 2}$$

$$\frac{Y(z)}{R(z)} = \frac{z^2}{3z^2 + z + 2}$$

## 2 Pulse T.F from block diagram

① open loop system



$$C(s) = G_2(s) E_2^* \quad (1) \xrightarrow{\text{starling}} C^*(s) = G_2^*(s) E_2^* \quad (1)^*$$

$$E_2(s) = R^* G_1(s) \quad ; \quad R^* = E_1^* \xrightarrow{\text{starling}} E_2^*(s) G_1^*(s) R^* \quad (2)^*$$

from  $(2)^*$  in  $(1)^*$

$$C^*(s) = G_1^*(s) \cdot G_2^*(s) \cdot R^*$$

$$\frac{C^*(s)}{R^*(s)} = G_1^*(s) \cdot G_2^*(s)$$

if we remove the sampler (2)



$$C(s) = G_1(s) \cdot G_2(s) R^*(s)$$

↓  
starling  $C^*(s) = \overline{G_1 G_2}^* R^*(s)$

we deal with  $G_1, G_2$  as one continuous signal

So  $\overline{G_1 G_2}^*(s)$  means starling of  $\overline{G_1 G_2}(s)$  (one signal)

$$\frac{C^*(s)}{R^*(s)} = \overline{G_1 G_2}^*(s)$$

$$\frac{C(z)}{R(z)} = \overline{G_1 G_2}(z)$$

Ex:



- ① Find T.F. =  $\frac{C(z)}{R(z)}$
- ② Find the unit step response

$$\frac{C(z)}{R(z)} = \overline{G_1 G_2(z)}$$

$$= z \left[ \frac{1 - e^{-Ts}}{s^2(s+1)} \right]$$

$$= (1 - z^{-1}) z \left[ \frac{1}{s^2(s+1)} \right]$$

$$= \left( \frac{z-1}{z} \right) \cdot z \left[ \frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+1} \right]$$

$$A_1 = 1, A_3 = 1, A_2 \parallel s=1 \Rightarrow \frac{1}{2} = A_1 + A_2 + \frac{A_3}{2}$$

$$\Rightarrow A_2 = -1$$

$$= \frac{z-1}{z} z \left[ \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right]$$

$\uparrow$        $\uparrow$        $\uparrow$   
 $t$        $1$        $e^{-t}$

$$= \left( \frac{z-1}{z} \right) \left[ \frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z \cdot e^{-T}} \right]$$

$$= \left( \frac{z-1}{z} \right) \left[ \frac{1}{(z-1)^2} - \frac{1}{z-1} + \frac{1}{z \cdot e^{-1}} \right]$$

$$\uparrow e^{-1} = 0.368$$

$$= \frac{z - 0.368 - (z-1)(z-0.368) + (z-1)^2}{(z-1)(z-0.368)}$$

$$\frac{C(z)}{R(z)} = \frac{0.368z + 0.264}{(z-1)(z-0.368)} \Rightarrow \text{أول مطلوب}$$

② unit step response  $\Rightarrow$  ثاني مطلوب

$$r(t) = 1$$

$$R(z) = \frac{z}{z-1}$$

$$C(z) = \left[ \frac{0.368z + 0.264}{(z-1)(z-0.368)} \right] \cdot R(z)$$

$$C(z) = \left[ \begin{array}{c} \downarrow \\ \dots \end{array} \right] \frac{z}{z-1}$$

$$C(z) = \frac{(0.368z + 0.264)z}{(z-1)^2(z-0.368)} \quad \downarrow \quad z^{-1} \cdot T$$

$$C(z) = z \left[ \frac{0.368z + 0.264}{(z-1)^2(z-0.368)} \right]$$

$$= z \left[ \frac{A_1}{(z-1)^2} + \frac{A_2}{(z-1)} + \frac{A_3}{(z-0.368)} \right]$$

$$\left| \begin{array}{l} A_1 = 1 \\ A_2 = -1 \\ A_3 = 1 \end{array} \right.$$

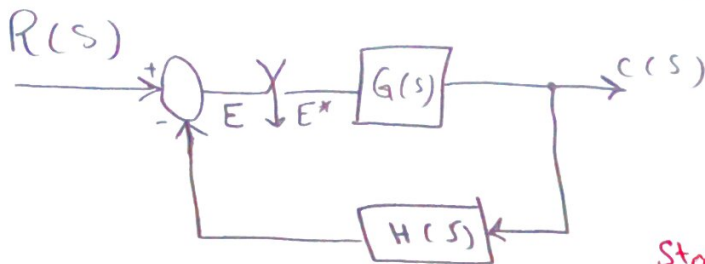
$$C(z) = \frac{z}{(z-1)^2} - \frac{z}{(z-1)} + \frac{z}{(z-0.368)} \quad \downarrow \quad z^{-1} \cdot T$$

$$c(k) = k - u(k) + (0.368)^k$$

$\Leftarrow$  Unit Step response



## ② close loop systems



① عمل سَمِية لافل وفرج ار Sampler موجود  
...  $E_3, E_2, E_1$

② هات معادلة الخرج الرئيسي بالاضافة

لمعادلات دخل ار Sampler

③ لو عندك نوع بيانات اعملها قبل ار Staring

④ العمل Staring

$$C(s) = G(s) E^*(s) \quad \text{--- ①}$$

$$E(s) = R(s) - H(s) \cdot C(s)$$

$$E(s) = R(s) - G H(s) E^*(s) \quad \text{--- ②}$$

$$C^*(s) = G^*(s) E^*(s) \quad \text{"Staring of 1"} \quad \text{①}^*$$

$$E^*(s) = R^*(s) - \overline{G H(s)}^* E^*(s) \quad \text{"Staring of 2"} \quad \text{②}^*$$

$$E^*(s) = \frac{R^*(s)}{1 + \overline{G H(s)}^*} \quad \text{③}$$

$$(1 + \overline{G H(s)}^*) E^*(s) = R^*(s)$$

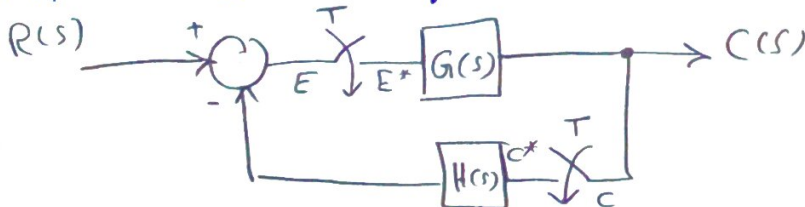
From ③ in ①<sup>\*</sup>

$$C^*(s) = \overline{G^*(s)} R^*(s)$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{G H(s)}^*}$$

$$\Rightarrow \frac{C(z)}{R(z)} = \frac{G(z)}{1 + G H(z)}$$

\* if we add sampler in feed back



$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z) H(z)}$$

\* Proof

$$C(s) = G(s) E^*(s) \xrightarrow{\text{star}} C^*(s) = G^*(s) E^*(s)$$

$$E = R - H(s) C(s) \xrightarrow{\text{star}} E^*(s) = R^* - H^* C^*(s)$$

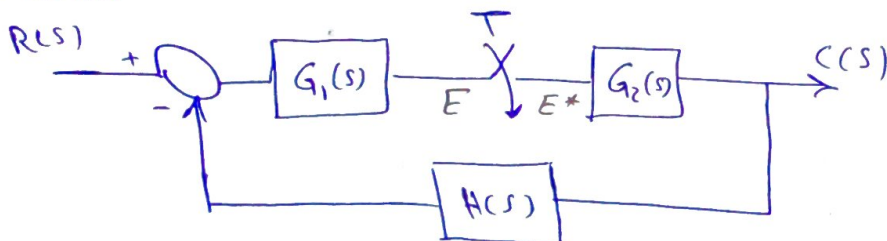
$$E^*(s) = R^* - H^* G^*(s) E^*(s)$$

$$\Rightarrow [1 + G^*(s) H^*(s)] E^*(s) = R^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1 + G^*(s) H^*(s)} \quad (3)$$

$$\text{From (3) in (1)} \Rightarrow \frac{C^*}{R^*} = \frac{G^*(s)}{1 + G^*(s) H^*(s)}$$

\* Ex 3



$$C(s) = G_2(s) E^*(s) \Rightarrow C^*(s) = G_2^*(s) E^*(s)$$

$$E^*(s) = G_1 [R(s) - G_2(s) H(s) E^*(s)]$$

$$= G_1 R(s) - G_1 G_2 H(s) E^*(s) \quad (2)$$

$$\Rightarrow E^*(s) = \overline{G_1 R(s)}^* - \overline{G_1 G_2 H(s)}^* E^*(s)$$

$\Rightarrow$  we can't find Pulse T.F.

Response  $\rightarrow$   $\frac{C(s)}{R(s)}$  و Impulse T.F  $\rightarrow$   $\frac{C^*(s)}{R^*(s)}$

$$(1 + \overline{G_1 G_2 H(s)}^*) E^*(s) = \overline{G_1 R(s)}^*$$

$$E^*(s) = \frac{\overline{G_1 R(s)}^*}{1 + \overline{G_1 G_2 H(s)}^*}$$

$\Rightarrow$  Continue

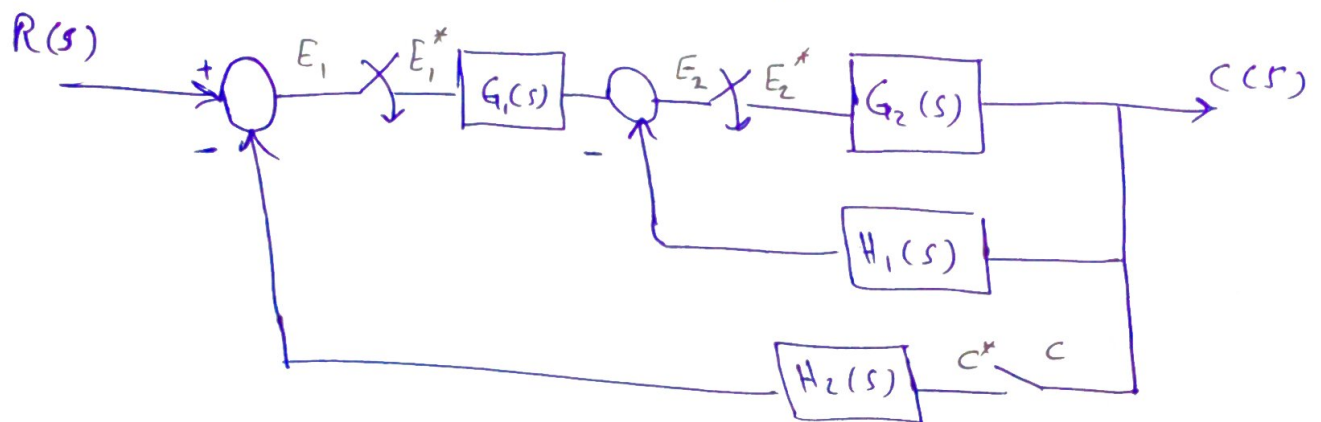
$$C^*(s) = \frac{G_2^* \overline{G_1 R^*(s)}}{1 + G_1 G_2 H(s)}$$

$$C(z) = \frac{G_2(z) \overline{G_1 R(z)}}{1 + G_1 G_2 H(z)}$$

The System output

Example :-

سوال امتحانہ وضوح



Signal flow graph

اتحاد زری اور

$$\frac{C(z)}{R(z)} = \frac{G_1(z) G_2(z)}{1 + G_2 H_1(z) + G_1 G_2 H_2(z)}$$

← اس کے لیے  
مساں نہ فقط

$$C(s) = G_2(s) E_2^*(s) \rightarrow (1)$$

$$E_2(s) = G_1(s) E_1^* - G_2 H_1 E_2^* \rightarrow (2)$$

$$E_1(s) = R(s) - H_2(s) C^*(s) \rightarrow (3)$$

$$C^*(s) = G_2^*(s) E_2^*(s) \rightarrow (1)^*$$

$$E_2^*(s) = G_1^*(s) E_1^*(s) - \overline{G_2 H_1}^* E_2^*(s) \rightarrow (2)^*$$

$$E_1^*(s) = R^*(s) - H_2^*(s) G_2^*(s) E_2^*(s) \rightarrow (3)^*$$



from (3)<sup>\*</sup> in (2)<sup>\*</sup>

$$E_2^*(s) = G_1^*(s) (R^*(s) - H_2^*(s) \cdot G_2^*(s) E_2^*(s)) - \overline{G_2 H_1}^*(s) E_2^*(s)$$

$$E_2^* = G_1^* R^* - G_1^* G_2^* H_2^* E_2^* - \overline{G_2 H_1}^* E_2^*$$

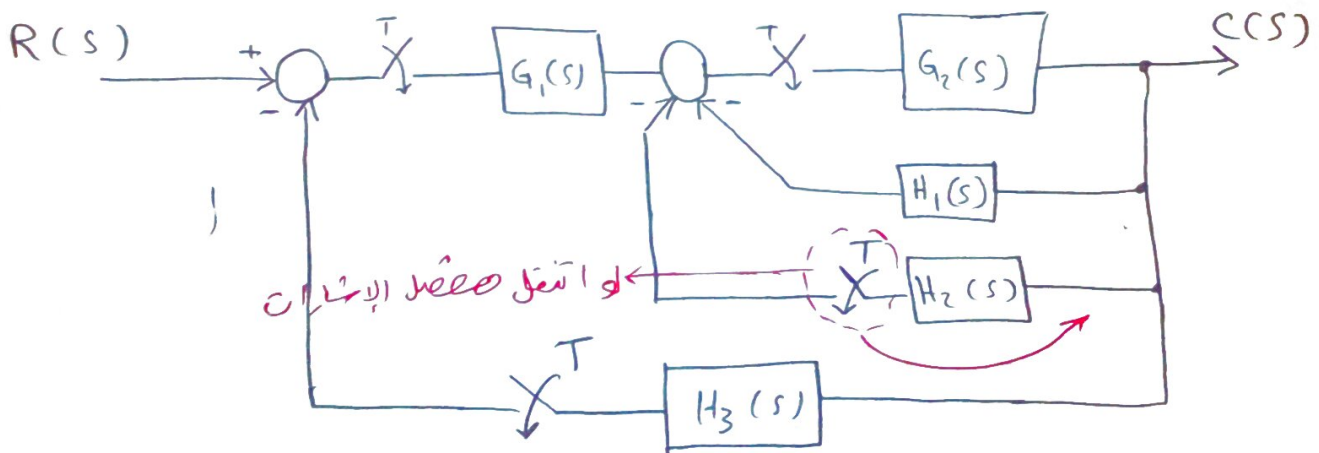
$$(1 + \overline{G_2 H_1}^*(s) + G_1^*(s) \cdot G_2^*(s) \cdot H_2^*(s)) E_2^*(s) = G_1^*(s) R^*(s)$$

$$\Rightarrow E_2^*(s) = \frac{G_1^*(s) \cdot R^*(s)}{1 + \overline{G_2 H_1}^*(s) + G_1^*(s) \cdot G_2^*(s) \cdot H_2^*(s)} \Rightarrow (4)$$

from (4) in (1)<sup>\*</sup>

$$\frac{C^*(s)}{R^*(s)} = \frac{G_1^*(s) G_2^*(s)}{1 + \overline{G_2 H_1}^*(s) + G_1^*(s) \cdot G_2^*(s) \cdot H_2^*(s)}$$

$$\Rightarrow \frac{C(z)}{R(z)} = \frac{G_1(z) \cdot G_2(z)}{1 + \overline{G_2 H_1}(z) + G_1(z) G_2(z) \cdot H_2(z)}$$



$$\frac{C(z)}{R(z)} = \frac{G_1(z) G_2(z)}{1 + \overline{G_2 H_1}(z) + \overline{G_2 H_2}(z) + G_1(z) \overline{G_2 H_3}(z)}$$